Integration

⇒ AS Recall

$$f(x) = \int f'(x)\,dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{n+1} \times \frac{1}{a} + c$$

→ Application

→ Area under curve $= \int_a^b f(x).dx$

→ Volume of revolution $= \pi \int_a^b y^2.dx$ or $\pi \int_a^b x^2.dy$

⇒ A level

$$\int \frac{1}{1+x^2} \, dx = \tan^{-1}(x) + c$$

$$\int \frac{1}{a^2+x^2} \, dx = \frac{1}{a} \tan^{-1}\left[\frac{x}{a}\right] + c$$

$$\int \frac{1}{a^2+(px+q)^2} = \frac{1}{a} \tan^{-1}\left[\frac{px+q}{a}\right] \times \frac{1}{p} + c$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1}(x) + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \frac{1}{a} \sin^{-1}\left[\frac{x}{a}\right]$$

Integration

⇒ Exponential Function

→ $\int e^x \, dx = e^x + c$

→ $\int \frac{1}{x} \, dx = \ln(|x|) + c$

→ $\int \frac{1}{x^2+1} \, dx = \tan^{-1}(x) + c$

→ $\int \frac{1}{x^2+a^2} \, dx = \frac{1}{a} \tan^{-1}\left[\frac{x}{a}\right] + c$

→ $\int e^{ax+b} \, dx = \frac{e^{ax+b}}{a} + c$

→ $\int \frac{1}{ax+b} \, dx = \frac{\ln(|ax+b|)}{a} + c$

→ $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{(n+1)(a)}$

⇒ Trigonometric Functions

→ $\int \sin x \, dx = -\cos x + c$

→ $\int \sin(ax+b) \, dx = \frac{-\cos(ax+b)}{a} + c$

→ $\int \cos x \, dx = \sin x + c$

→ $\int \cos(ax+b) \, dx = \frac{\sin(ax+b)}{a} + c$

$\rightarrow \int \sec^2 x \, dx = \tan x + c$

$\rightarrow \int \sec^2 (ax+b) \, dx = \frac{\tan(ax+b)}{a} + c$

$\Rightarrow$ Integrating $\sin^2 x$ and $\cos^2 x$

$\rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$

$\rightarrow \cos^2 x = \frac{1+\cos 2x}{2}$

$\Rightarrow$ Integrating $\tan^2 x$

$\rightarrow 1 + \tan^2 x = \sec^2 x$

$\Rightarrow \int \frac{f'(x)}{f(x)} = \ln(f(x)) + c$

$\Rightarrow \int \frac{k f'(x)}{f(x)} \, dx = k \ln(f(x)) + c$

$\Rightarrow$ Integration by parts

$u \int v \, dx - \int \left(\int v \right) \cdot \frac{du}{dx}$

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⇒ Integration by substitution

Q. $\int_0^{\frac{\pi}{4}} \frac{\sqrt{1+3\tan x}}{\cos^2 x}\, dx$

$$I = \int_0^{\frac{\pi}{4}} \sqrt{1+3\tan x}\ \sec^2 x \ dx$$

let $1+3\tan x = u$

$$3\sec^2 x = du$$
$$\sec^2 x = \frac{du}{3}$$

∴ finding new limits by subst of limits in u

$$\boxed{\begin{array}{ll} 1+3\tan(0) = 1 & x=0 \\ 1+3\tan\left(\frac{\pi}{4}\right) = 4 & x=\frac{\pi}{4} \end{array}}$$

New limits : 1 and 4

$$\int_1^4 \sqrt{u}\ \frac{du}{3}$$

$$I = \int_1^4 u^{1/2} \times \frac{1}{3}\ du$$

$$I = \int_1^4 \frac{u^{3/2}}{3/2} \times \frac{1}{3}$$

$$= \frac{2}{9}\left[4^{3/2} - 1^{3/2}\right] = \frac{2}{9} \times 7 = \boxed{\frac{14}{9}}$$

→) <u>Integration by parts</u>

$$\int uv \, dx = u\int v\,dx - \int\left(\int v\,dx \cdot \frac{du}{dx}\right) dx$$

$$\int \ln x \times x^{-\frac{1}{2}} = \cancel{\int x^{-\frac{1}{2}} \times \ln x}$$

<u>ILATE</u>

$$\ln x \int_{1}^{u} x^{-\frac{1}{2}}\,dx - \int\left(\int x^{-\frac{1}{2}}\,dx \times \frac{1}{x}\right) dx$$

$$\ln x \int \frac{x^{\frac{1}{2}}}{1/2} - 2x^{\frac{1}{2}} \times \frac{1}{x}$$